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High Gain Free Electron Lasers Driven by Flat Electron Beam

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Abstract

A variational solution of the fundamental eigenmode is presented for high gain free electron lasers driven by flat electron beam having unequal emittances, betatron focusings and beam sizes in two transverse planes.

Key words: flat beam, elliptical beam, high gain free electron laser

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1 Introduction

The main objective of this article is to expand our earlier work [1] on the eigenmode solution for high gain FELs into the situation where electron beam distribution is no longer axially symmetrical. Such a beam, generally termed flat beam, may have unequal emittances, betatron focusings, or beam sizes in two transverse planes. Earlier studies on this subject date back more than a

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decade ago. Using a special variational technique [2], the fundamental mode was calculated by Xie [3] for parallel beam having elliptical cross section. Later, a dispersion equation for the eigenvalue of the fundamental mode was derived using a different technique by Chin et al. [4] for more general cases including unequal emittances and betatron focusings. However, the latter technique was known at birth [4] to introduce a systematic error when approaching the 1D limit, thus compromising its accuracy for short wavelength FEL calculations, and furthermore, the dispersion equation was not solved.

In this article, we present the first general flat beam solution and specific calculations for the fundamental eigenmode and examine effects of flat beam on high gain FEL performance. The eigenmode equation is formulated for flat beam in section 2. The variational solution is derived in section 3. Finally, specific calculations for the LCLS case are given in section 4.

2 Eigenmode Equation

We specify the radiation field by a complex envelope $a_r(\mathbf{x}, z, t)$ slowly varying with respect to a carrier wave $\exp(k_r z - \omega_r t)$, where $\mathbf{x} \equiv \{x, y\}$, the carrier frequency $\omega_r = ck_r = 2\pi c/\lambda_r$ is determined by the resonance condition $k_r = 2\gamma_0^2 k_w/(1 + a_w^2)$, $\gamma_0 mc^2$ is average electron energy, $\lambda_w = 2\pi/k_w$ is wiggler period, $a_w = eB_{rms}\lambda_w/2\pi mc$, and B_{rms} is rms wiggler field assumed constant along wiggler axis. The radiation field is normalized according to $|a_r| = eE_{rms}/k_r mc^2$, where $E_{rms}(\mathbf{x}, z, t)$ is rms amplitude of the electric field. Upon introducing Fourier transform by

$$a_\nu = \frac{1}{\sqrt{2\pi}} \int d\theta e^{-i\nu\theta} (e^{i\theta} a_r),$$

where $\theta = (k_r + k_w)z - \omega_r t$ and $\nu = \omega/\omega_r$, the eigenmode of the form $a_\nu = a(\mathbf{x})\exp(-i\mu z)$ can be determined by the mode equation [1,5,6]

$$\left(\mu - k_w\Delta\nu + \frac{1}{2k_r}\frac{\partial^2}{\partial\mathbf{x}^2}\right)a(\mathbf{x}) = \frac{ih}{16k_w L_{1d}^3}\int\limits_{-\infty}^{\infty} d^2\mathbf{p}d\eta\frac{\partial F}{\partial\eta}\int\limits_{-\infty}^{0} dse^{-i(\mu-\xi)s}a(\mathbf{x}'), \quad (1)$$

with

$$\begin{aligned}
\xi &= 2k_w\eta - \frac{k_r}{2}(p_x^2 + k_{\beta x}^2x^2 + p_y^2 + k_{\beta y}^2y^2),\\
x' &= x\cos(k_{\beta x}s) + \frac{p_x}{k_{\beta x}}\sin(k_{\beta x}s),\\
y' &= y\cos(k_{\beta y}s) + \frac{p_y}{k_{\beta y}}\sin(k_{\beta y}s),\\
L_{1d} &= \frac{\lambda_w}{4\pi}\left(\frac{2\pi h\gamma_0^3}{r_e n_0\lambda_w^2 a_w^2 f_B^2}\right)^{\frac{1}{3}},
\end{aligned}$$

where μ is the complex eigenvalue related to our earlier notation [1] by $\mu = iq/2L_{1d} + k_w\Delta\nu$, L_{1d} is the 1D power gain length, $\Delta\nu = \nu - 1$, $\eta = (\gamma-\gamma_0)/\gamma_0$, $h = (2/\sqrt{3})^3$, $f_B = 1$ for helical wiggler and $f_B = J_0(\chi_w) - J_1(\chi_w)$ for planar wiggler, $\chi_w = a_w^2/2(1+a_w^2)$, r_e is the classical radius of electron, n_0 is the peak electron volume density on the axis, $F = F_\perp(\mathbf{x},\mathbf{p})F_\parallel(\eta)$ is unperturbed beam distribution function normalized by

$$\int\limits_{-\infty}^{\infty} d^2\mathbf{p}F_\perp(\mathbf{x}=0,\mathbf{p}) = 1, \quad \int\limits_{-\infty}^{\infty} d\eta F_\parallel(\eta) = 1.$$

Transverse focusing of the beam in wiggler is assumed to have a strength invariant along the axis, thus betatron motion is governed by

$$p_x = \frac{dx}{dz}, \quad \frac{dp_x}{dz} = -k_{\beta x}^2 x, \quad p_y = \frac{dy}{dz}, \quad \frac{dp_y}{dz} = -k_{\beta y}^2 y,$$

where $\beta_x = 1/k_{\beta x}$ and $\beta_y = 1/k_{\beta y}$ are constant betafunctions.

To perform specific calculations, we use a Gaussian model with

$$F_\perp = \frac{1}{2\pi\sigma_x\sigma_y k_{\beta x}k_{\beta y}} e^{-\frac{x^2+p_x^2/k_{\beta x}^2}{2\sigma_x^2}-\frac{y^2+p_y^2/k_{\beta y}^2}{2\sigma_y^2}}, \quad F_\parallel = \frac{1}{\sqrt{2\pi}\sigma_\eta} e^{-\frac{\eta^2}{2\sigma_\eta^2}},$$

where $\sigma_x = \sqrt{\beta_x\varepsilon_x}$ and $\sigma_y = \sqrt{\beta_y\varepsilon_y}$ are rms beam sizes matched to the focusing channel, ε_x and ε_y are rms emittances, and σ_η is relative rms energy spread. Using $\mathbf{X} \equiv \{X, Y\} \equiv \{x/\sigma_x, y/\sigma_y\}$ and $\tau = s/2L_{1d}$, Eq. (1) can be expressed in a more compact scaled form

$$\left(2\eta_{dx}\frac{\partial^2}{\partial X^2} + 2\eta_{dy}\frac{\partial^2}{\partial Y^2} + \bar{\kappa}\right) a(\mathbf{X}) = \int_{-\infty}^{\infty} d^2\mathbf{X}'\Pi(\mathbf{X},\mathbf{X}')a(\mathbf{X}'), \tag{2}$$

where $\bar{\kappa} = \kappa - \eta_\omega$, $\kappa = 2L_{1d}\mu$ and

$$\begin{aligned}
&\Pi(\mathbf{X},\mathbf{X}') = \int_{-\infty}^{0} \frac{\tau d\tau h e^{-\Psi}}{2\pi\Phi}, \\
&\Phi = \sin(2\sqrt{\eta_{dx}\eta_{\varepsilon x}}\tau)\sin(2\sqrt{\eta_{dy}\eta_{\varepsilon y}}\tau), \\
&\Psi = i\kappa\tau + 2\eta_\gamma^2\tau^2 + \frac{(1+i\eta_{\varepsilon x}\tau)\Omega_x}{2\sin^2(2\sqrt{\eta_{dx}\eta_{\varepsilon x}}\tau)} + \frac{(1+i\eta_{\varepsilon y}\tau)\Omega_y}{2\sin^2(2\sqrt{\eta_{dy}\eta_{\varepsilon y}}\tau)}, \\
&\Omega_x = X^2 + X'^2 - 2XX'\cos(2\sqrt{\eta_{dx}\eta_{\varepsilon x}}\tau), \\
&\Omega_y = Y^2 + Y'^2 - 2YY'\cos(2\sqrt{\eta_{dy}\eta_{\varepsilon y}}\tau).
\end{aligned}$$

There are six scaling parameters in Eq. (2): $\eta_{dx} = L_{1d}/2k_r\sigma_x^2$ and $\eta_{dy} = L_{1d}/2k_r\sigma_y^2$ are diffraction parameters; $\eta_{\varepsilon x} = 2L_{1d}k_r k_{\beta x}\varepsilon_x$ and $\eta_{\varepsilon y} = 2L_{1d}k_r k_{\beta y}\varepsilon_y$ characterize effective spread in longitudinal phase due to emittance and betatron focusing, and $\eta_\gamma = 2L_{1d}k_w\sigma_\eta$ due to energy spread, respectively; and $\eta_\omega = 2L_{1d}k_w\Delta\nu$ is a frequency detuning parameter. The 1D power gain length can now be expressed as

$$L_{1d} = \frac{\lambda_w}{4\pi}\left(\frac{hI_A\sigma_x\sigma_y k_w^2\gamma_0^3}{I_b a_w^2 f_B^2}\right)^{\frac{1}{3}},$$

where I_b is beam current and $I_A = 17.05$ kA is the Alfv̂en current.

3 Variational Solution

Next, we present an approximate solution for the fundamental mode. According to the recipe of a special variational technique [2], a variational functional may be constructed from Eq. (2) as

$$\int_{-\infty}^{\infty} d^2\mathbf{X}\, a(\mathbf{X}) \left\{ 2\eta_{dx}\frac{\partial^2}{\partial X^2} + 2\eta_{dy}\frac{\partial^2}{\partial Y^2} + \bar{\kappa} \right\} a(\mathbf{X}) = \int_{-\infty}^{\infty} d^2\mathbf{X} d^2\mathbf{X}' a(\mathbf{X}) \Pi(\mathbf{X}, \mathbf{X}')\, a(\mathbf{X}').$$

Substituting into the variational functional a trial solution of the form

$$a(\mathbf{X}) = \exp(-\alpha_x X^2 - \alpha_y Y^2),$$

where α_x and α_y are complex variational parameters to be determined, and applying the variational conditions [2]

$$\frac{\delta\kappa}{\delta\alpha_x} = 0, \quad \frac{\delta\kappa}{\delta\alpha_y} = 0,$$

to the resulting equation, we obtain three equations from which the eigenvalue κ and mode parameters α_x and α_y can be determined by

$$F_1 \equiv \frac{\bar{\kappa}}{4\sqrt{\alpha_x\alpha_y}} - \frac{\eta_{dx}}{2}\sqrt{\frac{\alpha_x}{\alpha_y}} - \frac{\eta_{dy}}{2}\sqrt{\frac{\alpha_y}{\alpha_x}} - \int_{-\infty}^{0} \frac{\tau d\tau h e^{-f_1}}{\sqrt{f_{2x}f_{2y}}} = 0, \tag{3}$$

$$F_2 \equiv -\frac{\bar{\kappa}}{8\alpha_x\sqrt{\alpha_x\alpha_y}} - \frac{\eta_{dx}}{4\sqrt{\alpha_x\alpha_y}} + \frac{\eta_{dy}}{4\alpha_x}\sqrt{\frac{\alpha_y}{\alpha_x}} + \int_{-\infty}^{0} \frac{\tau d\tau h e^{-f_1} f'_{2x}}{2f_{2x}\sqrt{f_{2x}f_{2y}}} = 0, \tag{4}$$

$$F_3 \equiv -\frac{\bar{\kappa}}{8\alpha_y\sqrt{\alpha_x\alpha_y}} - \frac{\eta_{dy}}{4\sqrt{\alpha_x\alpha_y}} + \frac{\eta_{dx}}{4\alpha_y}\sqrt{\frac{\alpha_x}{\alpha_y}} + \int_{-\infty}^{0} \frac{\tau d\tau h e^{-f_1} f'_{2y}}{2f_{2y}\sqrt{f_{2x}f_{2y}}} = 0, \tag{5}$$

where

$$F_2(\kappa,\alpha_x,\alpha_y) = \frac{\partial F_1(\kappa,\alpha_x,\alpha_y)}{\partial\alpha_x},$$
$$F_3(\kappa,\alpha_x,\alpha_y) = \frac{\partial F_1(\kappa,\alpha_x,\alpha_y)}{\partial\alpha_y},$$
$$f_1 = i\kappa\tau + 2\eta_\gamma^2\tau^2,$$
$$f_{2x} = (1+i\eta_{\varepsilon x}\tau)^2 + 4\alpha_x(1+i\eta_{\varepsilon x}\tau) + 4\alpha_x^2\sin^2(2\sqrt{\eta_{dx}\eta_{\varepsilon x}}\tau),$$
$$f_{2y} = (1+i\eta_{\varepsilon y}\tau)^2 + 4\alpha_y(1+i\eta_{\varepsilon y}\tau) + 4\alpha_y^2\sin^2(2\sqrt{\eta_{dy}\eta_{\varepsilon y}}\tau),$$
$$f'_{2x} = \frac{\partial f_{2x}}{\partial\alpha_x} = 4(1+i\eta_{\varepsilon x}\tau) + 8\alpha_x\sin^2(2\sqrt{\eta_{dx}\eta_{\varepsilon x}}\tau),$$
$$f'_{2y} = \frac{\partial f_{2y}}{\partial\alpha_y} = 4(1+i\eta_{\varepsilon y}\tau) + 8\alpha_y\sin^2(2\sqrt{\eta_{dy}\eta_{\varepsilon y}}\tau).$$

It is noted that in the limit of $\eta_{\varepsilon x} = \eta_{\varepsilon y} = \eta_\gamma = 0$, Eqs.(3,4,5) are reduced to the same equations studied earlier [3] for the case of parallel Gaussian beam with elliptical cross section.

Given parameter α_x and α_y, the mode properties can be determined completely by comparing the mode profile

$$a = \exp\left(-\frac{\alpha_x x^2}{\sigma_x^2} - \frac{\alpha_y y^2}{\sigma_y^2}\right),$$

with the usual Gaussian description

$$a = \exp\left(-\frac{x^2}{w_x^2} + \frac{ik_r x^2}{2R_x} - \frac{y^2}{w_y^2} + \frac{ik_r y^2}{2R_y}\right),$$

where w_x and w_y are mode sizes, R_x and R_y are radii of phasefront curvature. In particular, we have for mode sizes

$$w_x = \frac{\sigma_x}{\sqrt{\alpha_{xr}}}, \quad w_y = \frac{\sigma_y}{\sqrt{\alpha_{yr}}},$$

and for far field divergence angles

$$\theta_x = \sqrt{\alpha_{xr}\left(1+\frac{\alpha_{xi}^2}{\alpha_{xr}^2}\right)}\left(\frac{\lambda_r}{\pi\sigma_x}\right), \quad \theta_y = \sqrt{\alpha_{yr}\left(1+\frac{\alpha_{yi}^2}{\alpha_{yr}^2}\right)}\left(\frac{\lambda_r}{\pi\sigma_y}\right).$$

Finally, power gain length of the fundamental mode is related to the eigenvalue by $L_g = L_{1d}/\kappa_i$.

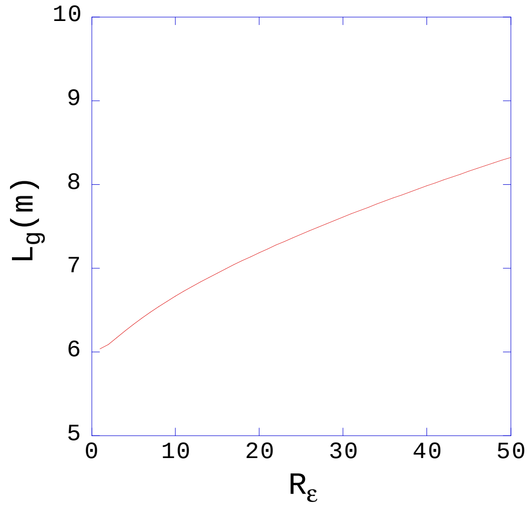

Fig. 1. L_g as a function of R_ε varied from 1 to 50 with β_x and β_y optimized to minimize L_g.

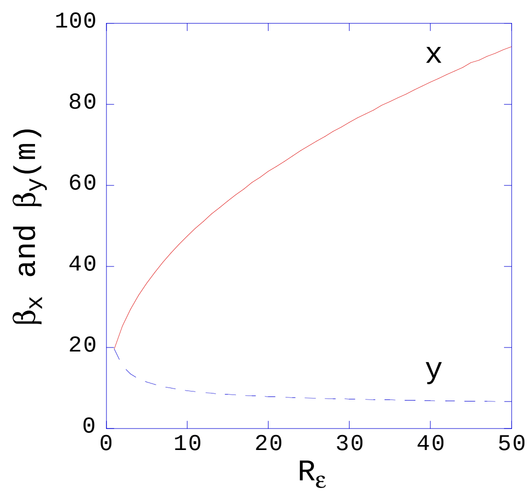

Fig. 2. Optimized β_x and β_y as functions of R_ε varied from 1 to 50.

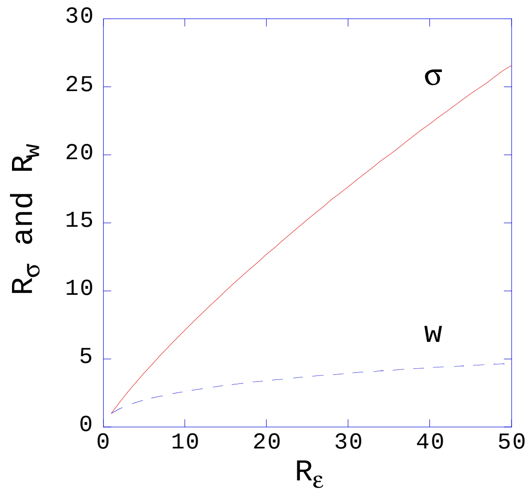

Fig. 3. R_σ and R_w as functions of R_ε varied from 1 to 50 with optimized β_x and β_y.

4 LCLS Examples

Given the solution of Eqs.(3,4,5), we are now ready to examine effects of flat beam on high gain FEL performance. Let's first define ratios of emittances, betafunctions, beam sizes and mode sizes respectively by

$$R_\varepsilon = \frac{\varepsilon_x}{\varepsilon_y}, \; R_\beta = \frac{\beta_x}{\beta_y}, \; R_\sigma = \frac{\sigma_x}{\sigma_y}, \; R_w = \frac{w_x}{w_y}.$$

Consider a scenario in which R_ε is increased from unity under the constraint that 4D emittance $\varepsilon_x \varepsilon_y = \varepsilon_0^2$ is kept constant. At each value of R_ε, L_g is minimized by varying β_x, β_y and frequency detuning. We shall take LCLS nominal design values for all other parameters [7]: $\lambda_r = 1.5$Å, $\gamma_0 = 28009$, $I_b = 3.4$kA, $\gamma_0\varepsilon_0 = 1.5$mm-mrad, $\sigma_\eta = 2 \times 10^{-4}$, a planar wiggler with $\lambda_w =$ 3cm and $\sqrt{2}a_w = 3.7$. In this scenario R_ε is the only free varying parameter.

Figure 1 shows L_g as a function of R_ε varied from 1 to 50 with optimized β_x and β_y given in Figure 2. At larger value of ε_x, focusing has to be relaxed with larger β_x to minimize gain reduction due to angular spread. On the other

Table 1

LCLS Examples

R_ε	1	10	50
$\gamma_0\varepsilon_x$ (mm-mr)	1.5	4.74	10.6
$\gamma_0\varepsilon_y$ (mm-mr)	1.5	0.474	0.212
β_x (m)	20	47	95
β_y (m)	20	9.4	6.7
σ_x (μm)	32	90	190
σ_y (μm)	32	13	7.1
w_x (μm)	44	76	120
w_y (μm)	44	29	26
θ_x (μr)	1.6	1.1	0.83
θ_y (μr)	1.6	2.4	3.1
L_g (m)	6.0	6.7	8.3
L_{1d} (m)	3.1	3.1	3.3

hand, focusing can be enhanced with smaller β_y at smaller value of ε_y. Figure 3 shows R_σ and R_w as functions of R_ε varied from 1 to 50. Notice the aspect ratio of laser mode is much less than that of electron beam. More details are given in Table 1 for three cases $R_\varepsilon = 1, 10, 50$.

5 Conclusions

We have presented an effective solution of the eigenmode for high gain FELs driven by general flat beam. It is found that the gain length increases with emittance ratio when 4D emittance is kept constant, and the rate of increase is rather weak in the LCLS parameter regime if beam focusing is simultaneously optimized. In addition, the aspect ratio of laser mode is much smaller than that of electron beam for larger emittance ratio. Effects of flat beam in other regimes and for other scenarios can be readily evaluated with the solution and methods provided here.

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